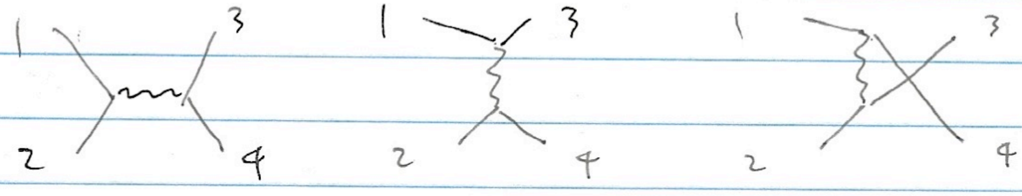
Schwartz

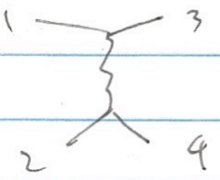
7.3 $$\mathcal{L} = -\frac{1}{2}\phi_e(\Box + m_e^2)\phi_e - \frac{1}{2}A_0\Box A_0 + e m_e A_0 \phi_e \phi_e .$$

(a) $e^- e^- \to e^- e^-$



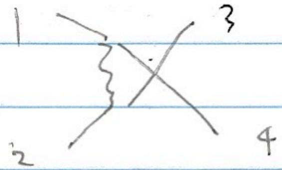
(b) $\Uparrow$ would be forbidden in real QED, because

charge is not conserved at the vertex

(c) 

2nd order.

$$(ie m_e)^2 \frac{i}{k^2+i\varepsilon} \delta(p_1-p_3-k) \cdot \delta(p_2-p_4+k)$$



$$(ie m_e)^2 \frac{i}{k^2+i\varepsilon} \delta(p_1-p_4-k) \, \delta(p_2-p_3+k)$$

$$M_1 = \int \frac{d^4k}{(2\pi)^4} (ie m_e)^2 \frac{i}{k^2+i\varepsilon} \delta(p_1-p_3-k)\, \delta(p_2-p_4+k)$$

$$= (ie m_e)^2 \frac{i}{(p_1-p_3)^2+i\varepsilon} \delta(p_2-p_3+p_1-p_3)$$

$$= \boxed{(ie m_e)^2 \frac{i}{t+i\varepsilon} \delta(\Sigma p)}$$

$t=(p_1-p_3)^2$

$$M_2 = \int \frac{d^4k}{(2\pi)^4} (ie m_e)^2 \frac{i}{k^2+i\varepsilon} \delta(p_1-p_4-k)\, \delta(p_2-p_3+k)$$

$$= (ie m_e)^2 \frac{i}{(p_1-p_4)^2+i\varepsilon} \delta(p_2-p_3+p_1-p_4)$$

$$= \boxed{(ie m_e)^2 \frac{i}{u+i\varepsilon} \delta(\Sigma p)}$$

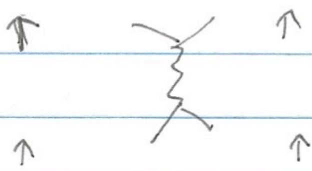
$u=(p_1-p_4)^2$

$$M_1+M_2 = (ie m_e)^2 i \delta(\Sigma p) \left[\frac{1}{t} - \frac{1}{u} \right]$$

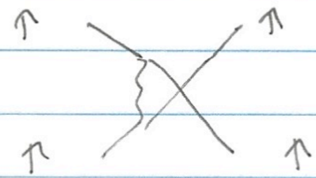
(d) Only possible spin initial and final states:

$|\uparrow\uparrow\rangle \to |\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle \to |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle \to |\downarrow\downarrow\rangle$

$|\uparrow\uparrow\rangle \to |\uparrow\uparrow\rangle$:

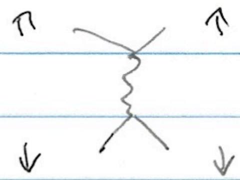


,



Same for $|\downarrow\downarrow\rangle \to |\downarrow\downarrow\rangle$

$|\uparrow\downarrow\rangle \to |\uparrow\downarrow\rangle$:



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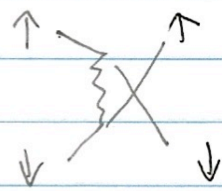
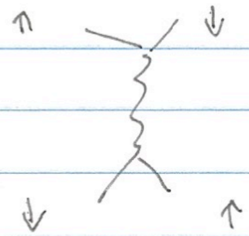
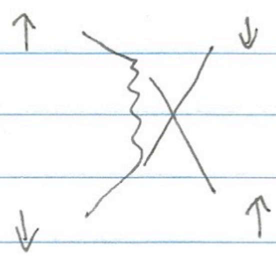


Is forbidden

$|\uparrow\downarrow\rangle \to |\downarrow\uparrow\rangle$:



forbidden



e) $|\uparrow\uparrow\rangle \to |\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle \to |\downarrow\downarrow\rangle$ gets contribution from t, u channel,

$|\uparrow\downarrow\rangle \to |\uparrow\downarrow\rangle$ gets t channel, $|\uparrow\downarrow\rangle \to |\downarrow\uparrow\rangle$ gets u channel.